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PREDICTION OF MEAN AND DESIGN FATIGUE LIVES OF STEEL FIBROUS CONCRETE USING S-N RELATIONSHIPS

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The paper presents a study on the fatigue strength of steel fibre reinforced concrete (SFRC) containing different volume fractions, aspect ratios and types of steel fibres, for various levels of the fatigue stress. The fatigue test data available in literature has been used for analysis. The test data is used to generate the S-N curves and an Equation is proposed by regression analysis to predict the flexural fatigue strength of SFRC. A probabilistic approach is used to predict the fatigue reliability of SFRC. The fatigue-life distributions of SFRC at a given stress level, is shown to approximately follow the two-parameter Weibull distribution. The S-N relationships have been used to obtain the parameters of the Weibull distribution. Mean and Design fatigue lives have been computed for different stress levels for SFRC with different combinations of fibres, corresponding to different probabilities of failure.

INTRODUCTION

Considerable interest has developed in the fatigue strength of concrete members in recent years. There are several reasons for this. Firstly, the use of high strength materials require that the concrete members perform satisfactorily under high stress levels. Hence, the study of the effects of repeated loads on bridge slabs and crane beams is a matter of concern. Secondly, different concrete systems such as prestressed concrete railroad ties and continuously reinforced concrete pavement slabs are often used .The use of these systems demand a high performance product with an assured fatigue-life. Thirdly, there is a new recognition of the effects of repeated loading on a member, even if it does not cause a fatigue failure. There may be inclined cracks in the prestressed concrete beams at lower loads due to fatigue loading and the static load carrying capacity of the component material may be altered.

In a conventionally reinforced structure/element subjected to bending moment, fatigue failure may occur either in tension steel or in the compression zone of the concrete. However, since the highway and airfield pavements are usually unreinforced, the concrete is called upon to resist tension in bending. Majority of the research reported in literature on fatigue of plain as well as steel fibre reinforced concrete (SFRC) has focussed attention on

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flexural fatigue and to some extent on compression fatigue.

A number of research investigations were carried out to look into the fatigue behaviour of plain concrete since Feret's pioneer tests [1]. Many researchers [2-4] adopted a relationship between stress level S, which is the ratio of maximum fatigue stress f_{max} to the modulus of rupture f_r and the number of loading cycles N which cause failure. The relationship established is known as the Wholer Equation given below:

$$S = \frac{f_{max}}{f_r} = a + b \log_{10}(N) \tag{1}$$

where 'a' and 'b' are experimental coefficients. Oh [5] obtained the values of coefficients 'a' and 'b' for plain concrete. It was further shown that the statistical distribution of fatigue-life of plain concrete can approximately be described by two-parameter Weibull distribution [5-6]. The parameters of the Weibull distribution i.e. shape parameter 'α' and characteristic extreme life 'u' were obtained by different methods. Another form of the fatigue Equation used by researchers [4,5,7,8] is a modification of the Wholer Equation which incorporates a stress ratio R, which is the ratio of minimum fatigue stress f_{min} to the maximum fatigue stress f_{max} into the Wholer Equation. The modified Equation takes the following form:

$$S = \frac{f_{max}}{f_r} = 1 - \beta(1 - R) \log_{10}(N) \tag{2}$$

where β is experimental coefficient. This Equation can be used for $0 \leq R \leq 1$ but not for stresses which alternate between compression and tension. For Equation (2) to be valid, the S-N curves should not be based on measurements where the amplitude or the lower stress f_{min} is kept constant, but on a constant stress ratio $R = f_{min}/f_{max}$. Aas-Jakobsen [9] obtained the value of β in Equation (2) Equal to 0.064 for compression fatigue of concrete. However, Tepfers et al [7] recommended the value of β as 0.0685. Oh [5] tested the Equation (2) for flexural fatigue of plain concrete and obtained the value of β as 0.0690.

A few experimental investigations have been carried out to study the fatigue behaviour of steel fibre reinforced concrete. However, specimen sizes, loading conditions and fatigue failure criteria have varied over a wide range. Most of these studies on steel fibre reinforced concrete were mainly confined to the determination of its flexural fatigue endurance limit for different type/volume fraction/aspect ratio of fibres [10-14] although some studies focussed attention on studying other aspects of fibre reinforced concrete. Yin et al. [15] studied the fatigue behaviour of steel fibre reinforced concrete under uniaxial and biaxial compression and observed that the S-N curves can be approximated by two straight lines connected by a curved knee instead of a single straight line. Ramakrishnan et al. [16-18] studied the flexural fatigue strength of fibre reinforced concrete and proposed constitutive relations and models to predict the fatigue strength of this material. A summary of mechanical models is given for numerically simulating the fatigue behaviour of fibre reinforced concrete.

RESEARCH SIGNIFICANCE

Johnston and Zemp [14] reported the results of fatigue tests on SFRC specimens containing different types, volume fractions and aspect ratios of steel fibres. They studied the performance of SFRC by developing S-N curves and the endurance limits were obtained for various combinations of steel fibres. As the fatigue life data of SFRC, even at a particular stress level, show considerable variation due to random orientation of the fibres, little attention has been paid to the probabilistic analysis of fatigue life data of steel fibre reinforced concrete. Hence the work of Johnston and Zemp [14] provides good opportunity to apply the concepts of probabilistic analysis to their fatigue test data and to examine the two-parameter Weibull distribution for SFRC. The objectives of this paper is first to determine the fatigue strength of steel fibre reinforced concrete subjected to flexural fatigue loading and, second, to implement the concepts of probabilistic analysis to study the fatigue characteristics of SFRC. The two-parameter Weibull distribution has been examined to describe the fatigue behaviour of SFRC. A method of obtaining the distribution parameters from the S-N relationship is discussed. Mean and design fatigue lives are obtained from the S-N relationships for steel fibre reinforced concrete containing fibres of different characteristics.

FATIGUE STRENGTH AND S-N RELATIONSHIPS

Johnston and Zemp [14] presented the fatigue test data of steel fibre reinforced concrete under flexural loading. The fibre volume fractions were kept as 0.5%, 1.0% and 1.5%. Different types of steel fibres, i.e. smooth uniform wire (SW), surface-deformed wire (SDW), melt extract (ME) and slit sheet (SS), with different aspect ratios ranging from 47 to 100 were used in the investigation. The fatigue tests were carried out at various stress levels ranging from 0.99 to 0.75. The basic concrete initially chosen comprised 13mm gravel coarse aggregate, coarse sand (air-dried prior to batching), and 297 kg/m^3 of normal cement with conventional water-reducing and air-entraining admixtures. The complete fatigue life data of steel fibre reinforced concrete as obtained by Johnston and Zemp [14] is presented in Table 1.

The S-N curves represent the relationship between the fatigue stress level 'S' and number of load repetitions 'N' which cause failure of the specimen. This relationship is given by the Equation (1). Figure 1 summarizes the test results in the form of S-N curves obtained from this study for SFRC containing steel fibres of different characteristics. Linear regression is carried out based on the least squares method to determine the values of coefficients 'a' and 'b'. The values of coefficients obtained for fatigue life data of SFRC with different combinations of steel fibres are listed in Table 2. The material coefficient β in Equation (2) can not be obtained from the test data since the testing has been carried out by keeping f_{min} as constant and not stress ratio R as already mentioned. Therefore, Equation (1) can be used to predict the flexural fatigue strength of SFRC by using the values of the material coefficients as determined above for SFRC with different combinations of steel fibres.

Table 1. Flexural fatigue life data of steel fibre reinforced concrete [14]

0.5 percent SW(75)		**1.0 percent SW(75)**		**1.5 percent SW(75)**		**1.0 percent SW(50)**		**1.0 percent SDW(47)**		**1.0 percent ME(54)**		**1.0 percent SS(71)**	
S	**N**	**S**	**N**	**S**	**N**	**S**	**N**	**S**	**N**	**S**	**N**	**S**	**N**
0.95	190 290 580 800 960 1420 1820	0.99	80 100 170 250 690 700 1100	0.94	100 150 200 250 300 510 1300	0.93	200 230 560 640 1310 1410 2700	0.90	170 190 350 450 1110 1200 2600	0.90	810 1160 2300 2900 3000 3250 5360 6300	0.94	520 690 920 950 1910 2230
0.90	1300 2350 2560 4200 6550 8460 10210 23450	0.94	1100 2200 3100 3300 3500 8990 9300	0.89	1200 1300 2400 2600 3400 5000 7000	0.85	7480 9200 10850 12100 17110 22000	0.83	4500 7100 8000 8900 8950 14000 15900	0.83	11900 21800 39800 41000 59600 80000	0.89	3900 6800 9200 10200 21500 24600
0.82	55000 119000 201750 249000 330000 500000	0.96	28200 63500 72200 86100 195000 288000	0.81	16000 33000 35000 61000 85200 95000 200000	0.77	35200 101000 130000 185000 190000	0.75	33000 36000 62000 77400 97500 161000 242000	0.75	185000 220000 264000 278400 403000 500000	0.82	27000 57400 74000 128000 170900 270400

S is the stress level as a percentage of static flexural strength; N is the number of cycles to failure; SW is smooth wire; SDW is surface-deformed wire; ME is melt extract, and SS is slit sheet.

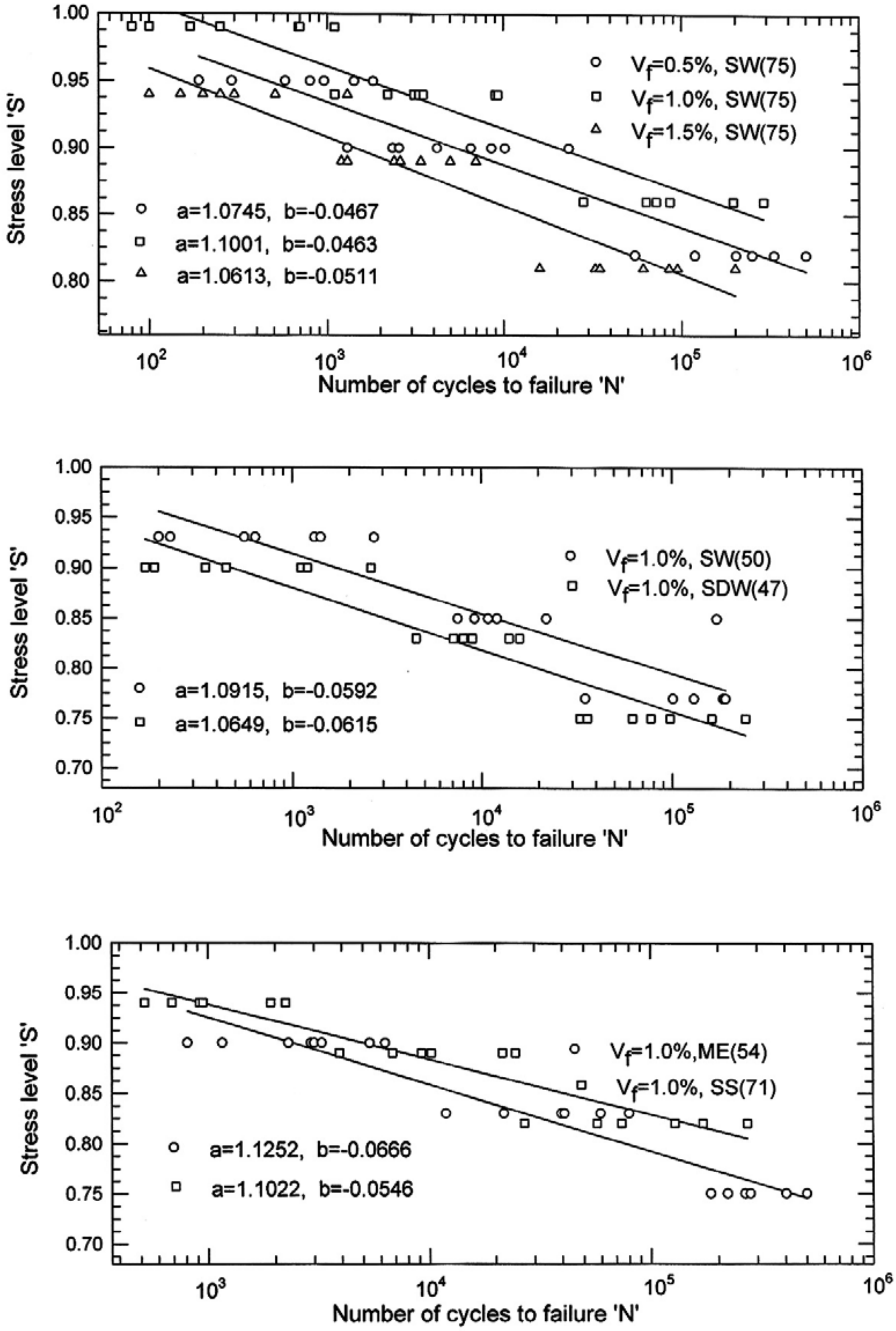


Figure 1. S-N Relationships for steel fibrous concrete containing fibres of different characteristics

Table 2. Coefficients ‘a’ and ‘b’ of Equation (1) for steel fibre reinforced concrete

Fibre Characteristics			**Coefficient ‘a’**	**Coefficient ‘b’**
Volume Fraction	**Aspect Ratio**	**Type**		
0.5%	75	Smooth Wire (SW)	1.0745	-0.0467
1.0%	75	Smooth Wire (SW)	1.1001	-0.0463
1.5%	75	Smooth Wire (SW)	1.0613	-0.0511
1.0%	50	Smooth Wire (SW)	1.0915	-0.0592
1.0%	47	Surface Deformed Wire (SDW)	1.0649	-0.0615
1.0%	54	Melt Extract (ME)	1.1252	-0.0666
1.0%	71	Slit Sheet (SS)	1.1022	-0.0546

FATIGUE-LIFE DISTRIBUTIONS OF SFRC

A number of mathematical models have been employed for the statistical description of fatigue data. One of the popular models has been the logarithmic-normal distribution function [20]. However, it was pointed out by Gumble [21] that the hazard function of this distribution decreases with increasing life, which violates the physical phenomenon of fatigue failure of materials. Thus because of physically valid assumptions and sound experimental verification, the Weibull distribution is most commonly used for the statistical description of fatigue data these days.

The survivorship function, $L_N(n)$, of two-parameter Weibull distribution may be written as follows [5, 6, 22, 23]:

$$L_N(n) = \exp\left[-\left(\frac{n}{u}\right)^{\alpha}\right] \tag{3}$$

Taking the logarithm twice of both sides of Equation (3)

$$\ln\left[\ln\left(\frac{1}{L_N}\right)\right] = \alpha\ln(n) - \alpha\ln(u) \tag{4}$$

Equation (4) represents a linear relationship between ln $[\ln(1/L_N)]$ and ln (n). This Equation can be used to verify whether the statistical distribution of fatigue life follows the two parameter Weibull distribution. In order to obtain a graph from Equation (4), the fatigue-life data at a given stress level must be first arranged in ascending order. The empirical survivorship function can be calculated from the following relation [5, 6, 19, 23]:

$$L_N = 1 - \frac{i}{k+1} \tag{5}$$

in which i = failure order number and k = number of fatigue data or sample size at a given stress level. A graph is plotted between ln [ln ($1/L_N$)] and ln (N), and a best fit line can be drawn through the data points by method of least squares. If a linear trend is observed for the fatigue-life data at a given stress level S, it can be assumed that the two-parameter Weibull distribution is a reasonable assumption for the statistical description of fatigue life data at that stress level. The parameters of the Weibull distribution α and u can be obtained from regression coefficients.

Figure 2 shows the plot of the fatigue life data of SFRC with smooth wire (SW) steel fibres of 0.5% volume fraction and aspect ratio of 75. The best fit line through the data points is drawn by regression analysis. The approximate straight line plots with correlation coefficient values exceeding 0.90 show that the fatigue life data at the corresponding stress levels can be described by the two-parameter Weibull distribution. The values of the parameters obtained are α = 1.1445 and u = 1023 for S = 0.95; α = 1.0107 and u = 8185 for S = 0.90 and α = 1.1671 and u = 290800 for S = 0.82. It has been observed that the fatigue life data of SFRC for other combinations of fibres as given in Table 1 can also be modeled by the two-parameter Weibull distribution. The detailed results are reported elsewhere [24]. The attention in this paper is drawn, however, to the next section.

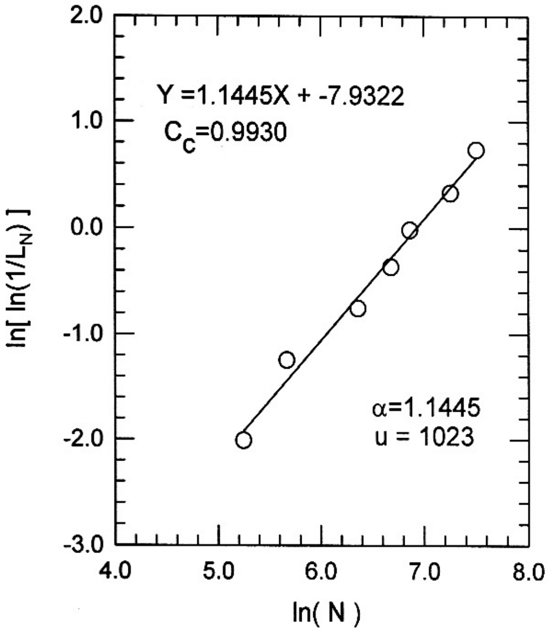
PARAMETERS FROM S-N RELATIONSHIPS

As shown in the previous section, the distribution parameters for a probability law can be obtained for a given stress level and the distribution of fatigue life of SFRC (at a given stress level) can be described by the two-parameter Weibull distribution. However, there is another method of obtaining the distribution parameters i.e. from the S-N relations. This method is based on an approximate assumption of constant variance for all the stress levels. For this, the following S-N relation may be assumed [5, 25]:

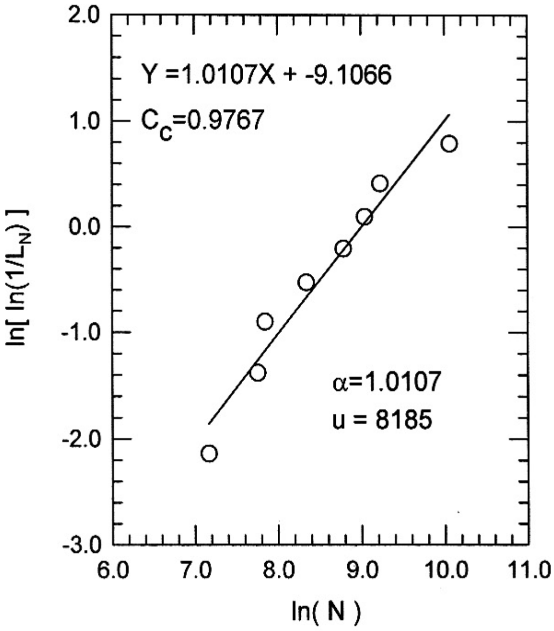
$$N\left(\frac{f_{max}}{f_r}\right)^m = C \tag{6}$$

in which m and C are empirical constants. Taking logarithm of both sides of the Equation (6)

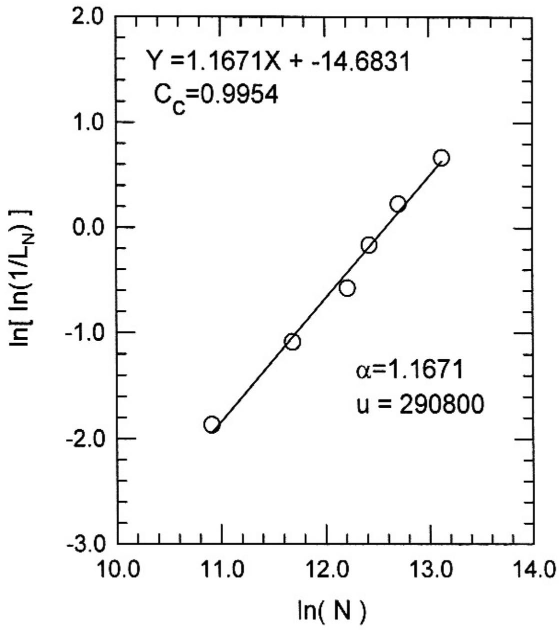
$$\log_{10}(N) = \log_{10} C - m \log_{10}\left(\frac{f_{max}}{f_r}\right) \tag{7}$$



(a). For stress level S=0.95



(b). For stress level S=0.90



(c). For stress level S=0.82

Figure 2. Regression analysis of fatigue life data for SFRC, SW(75), V_f=0.5%

$$Y = a + bX \tag{8}$$

in which $Y = \log_{10}(N)$; $X = \log_{10}\left(\frac{f_{max}}{f_r}\right)$; $a = \log_{10}C$; and $b = -m$.

If the fatigue life N, is assumed to follow the Weibull distribution (as has been shown in the previous section), the parameters of the distribution i.e. α and u may be determined from the following expressions [5, 25]:

$$\alpha^2 = \frac{\pi^2}{6(s)^2} \tag{9}$$

and

$$\ln(u) = \frac{0.5772}{\alpha} + \ln\left[C\left(\frac{f_{max}}{f_r}\right)^{-m}\right] \tag{10}$$

where s = estimate of the standard deviation or standard error of estimate of Y given X. The mean fatigue life for a given stress level may be obtained from the following relation [5]:

$$E[N] = \mu_N = C\left(\frac{f_{max}}{f_r}\right)^{-m} \exp\left[\frac{0.5772}{\alpha}\right] T\left(1 + \frac{1}{\alpha}\right) \tag{11}$$

in which T( ) is the gamma function and E[N] is the mean fatigue life.

The design fatigue life N_D, should be selected such that there is only a small probability that a fatigue failure will occur. Once the distribution function is determined, the design fatigue life may be selected corresponding to an acceptable reliability. The design reliability may be expressed as $P[N < N_D] = 1 - p_f$, where p_f is the probability of failure. Noting that $p_f = P[N < N_D]$, the design fatigue life corresponding to a permissible probability p_f is determined from the following relation.

$$N_D = u\left[\ln\frac{1}{1 - p_f}\right]^{\frac{1}{\alpha}} \tag{12}$$

The fatigue life data as described in the previous sections have been analyzed using Equations.(6), (7) and (8) and the following relations are obtained:

$$N\left(\frac{f_{max}}{f_r}\right)^{38.56} = 90.72 \quad \text{for SFRC SW(75), } V_f = 0.5\% \tag{13}$$

$$N\left(\frac{f_{max}}{f_r}\right)^{40.55} = 226.65 \text{ for SFRC SW(75), } V_f = 1.0\% \qquad (14)$$

$$N\left(\frac{f_{max}}{f_r}\right)^{35.18} = 36.81 \text{ for SFRC SW(75), } Vf = 1.5\% \qquad (15)$$

$$N\left(\frac{f_{max}}{f_r}\right)^{26.99} = 112.94 \text{ for SFRC SW(50), } Vf = 1.0\% \qquad (16)$$

$$N\left(\frac{f_{max}}{f_r}\right)^{27.00} = 40.20 \text{ for SFRC SDW(47), } V_f = 1.0\% \qquad (17)$$

$$N\left(\frac{f_{max}}{f_r}\right)^{25.93} = 196.71 \text{ for SFRC ME(54), } V_f = 1.0\% \qquad (18)$$

$$N\left(\frac{f_{max}}{f_r}\right)^{32.41} = 172.41 \text{ for SFRC SS(71), } V_f = 1.0\% \qquad (19)$$

The calculated values of the standard error of estimate of Y given X for the fatigue life of SFRC for various combinations of fibres are listed in Table 3. The shape parameter α and characteristic value u can be obtained from Equations. 9 and 10, respectively, and the calculated values are listed in Table 3. Equation 11 can be used to calculate the mean fatigue lives E[N] of SFRC, and the values thus calculated are listed in Tables 4–10 for various combinations of fibres. These tables also present design fatigue lives calculated using Equation 12, corresponding to various acceptable probabilities of failure p_f for various stress levels. Smaller acceptable probabilities of failure or higher reliabilities require the design lives to be small. It is noted here that the overall distribution parameters obtained from the S-N relationships may differ from those obtained in the previous section which are based on fatigue life data at a particular stress level.

Table 3. Values of weibull parameters obtained from S-N relationships.

Fibre Characteristics	**Standard Error of Estimate s**	**Shape Parameter α**	**Stress Level S**	**Characteristic Life u**
V_f = 0.5%, Aspect Ratio = 75, Smooth Wire (SW)	1.0578	1.2125	0.95	1055
			0.90	8486
			0.82	307238
V_f = 1.0%, Aspect Ratio = 75, Smooth Wire (SW)	1.0937	1.1727	0.99	557
			0.94	4557
			0.86	167975
V_f = 1.5%, Aspect Ratio = 75, Smooth Wire (SW)	1.0170	1.2611	0.94	513
			0.89	3508
			0.81	96373
V_f = 1.0%, Aspect Ratio = 50, Smooth Wire (SW)	0.9723	1.3191	0.93	1241
			0.85	14063
			0.77	202684
V_f = 1.0%, Aspect Ratio = 47, Surface Deformed Wire (SDW)	0.9573	1.3397	0.90	1063
			0.83	9461
			0.75	145943
V_f = 1.0%, Aspect Ratio = 54, Melt Extract (ME)	0.9149	1.4019	0.90	4563
			0.83	37249
			0.75	515903
V_f = 1.0%, Aspect Ratio = 71, Slit Sheet (SS)	0.8688	1.4763	0.94	1892
			0.89	11121
			0.82	158180

Table 4. Mean and design fatigue lives for SFRC, SW (75), V_f=0.5% (α=1.2125).

$S = f_{max} / f_r$		0.95	0.90	0.82
E[N]		1024	8234	298139
Design Fatigue Lives N_D				
	0.01	101	809	29307
P_f	0.05	231	1861	67386
	0.10	334	2688	97331

Table 5. Mean and design fatigue lives for SFRC, SW (75), V_f=1.0% (α=1.1727).

$S = f_{max} / f_r$		0.99	0.94	0.86
E[N]		542	4437	163538
Design Fatigue Lives N_D				
	0.01	79	646	23822
P_f	0.05	158	1291	47592
	0.10	214	1753	64606

Table 6. Mean and design fatigue lives for SFRC, SW (75), V_f=1.5% (α=1.2611).

$S = f_{max} / f_r$		0.94	0.89	0.81
E[N]		496	3395	93282
Design Fatigue Lives N_D				
	0.01	27	186	5107
P_f	0.05	77	526	14462
	0.10	122	834	22901

Table 7. Mean and design fatigue lives for SFRC, SW (50), V_f=1.0% (α=1.3191).

$S = f_{max} / f_r$		0.93	0.85	0.77
E[N]		1199	13587	195830
Design Fatigue Lives N_D				
	0.01	132	1496	21559
P_f	0.05	292	3309	47693
	0.10	415	4699	67724

Table 8. Mean and design fatigue lives for SFRC, SDW (47), V_f=1.0% (α=1.3397).

$S= f_{max} / f_r$		0.90	0.83	0.75
E[N]		1027	9136	140941
Design Fatigue Lives N_D				
	0.01	60	538	8296
P_f	0.05	167	1485	22915
	0.10	261	2327	35891

Table 9. Mean and design fatigue lives for SFRC, ME (54), V_f=1.0% (α=1.4019**).**

$S= f_{max} / f_r$		0.90	0.83	0.75
E[N]		4398	35902	497253
Design Fatigue Lives N_D				
	0.01	614	5014	69440
P_f	0.05	1250	10240	141324
	0.10	1711	13965	193420

Table 10. Mean and design fatigue lives for SFRC, SS (71), V_f=1.0% (α=1.4763).

$S= f_{max} / f_r$		0.94	0.89	0.82
E[N]		1823	10716	152413
Design Fatigue Lives N_D				
	0.01	242	1421	20217
P_f	0.05	501	2946	41907
	0.10	691	4065	57822

CONCLUSIONS

1. The test data is used to generate the S-N curves and Equations are proposed by regression analysis to predict the flexural fatigue strength of SFRC.
2. A probabilistic approach is employed to predict the fatigue reliability of SFRC. The fatigue-life distributions of SFRC at a given stress level, is shown to approximately follow the two-parameter Weibull distribution.
3. A method of obtaining the distribution parameters of the Weibull distribution from the S-N relationships is presented.
4. The parameters such as mean fatigue lives, characteristic value and design fatigue lives have been determined for various stress levels corresponding to different probabilities of failure for SFRC with different combinations of fibres.

NOTATIONS

f_{max} = Maximum fatigue stress
f_{min} = Minimum fatigue stress
f_r = Static flexural stress
S = Stress level = f_{max}/f_r
R = Stress ratio = f_{min}/f_{max}
L_N = Survivorship function or reliability function.
N = Number of cycles to failure or fatigue life
n = Specific value of N
N_D = Design fatigue life
u = Characteristic life or scale parameter of Weibull distribution
CV = Coefficient of variation of the data sample at a given stress level
α = Shape parameter of Weibull distribution
$T(\,)$ = Gamma function
$E[N]$ = Mean fatigue life
p_f = Probability of failure

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